



Calculus AB, section 9.1 – Solving Differential Equations using the Separation of Variables method

Example 1

Find the general solution to $y \frac{dy}{dx} - x = 0$ by the separation of variables method.

$$y \frac{dy}{dx} = x \qquad y^2 = x^2 + C$$

$$\int y \, dy = \int x \, dx \qquad y = \pm \sqrt{x^2 + C}$$

$$\frac{1}{2} y^2 = \frac{1}{2} x^2 + C$$

general solution

Find a particular solution if $y(0) = 3$.

$$3 = \sqrt{0^2 + C} \qquad y = \sqrt{x^2 + 9}$$

$$\frac{dy}{dx} = \frac{1}{2} (x^2 + 9)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2 + 9}}$$

$$y \left(\frac{x}{\sqrt{x^2 + 9}} \right) - x = 0 \qquad \cancel{\sqrt{x^2 + 9} \cdot \frac{x}{\sqrt{x^2 + 9}} - x = 0}$$

Example 2 – Initial Value Problem

Solve the initial value problem $y' = -ty$ with $y(0) = 3$.

$$\int \frac{dy}{y} = - \int t \, dt$$

$$| \quad | \quad | \quad - \quad - \quad \frac{1}{2} t^2 + C$$

$$\ln |y| = -\frac{1}{2}t^2 + C$$

$$y = e^{-\frac{1}{2}t^2 + C} = ce^{-\frac{1}{2}t^2}$$

general

$$3 = ce^0 \quad c = 3$$

$$y = 3e^{-\frac{1}{2}t^2}$$

Example 3

Solve: $\sqrt{1-x^2} y' = xy$ $\frac{dy}{y} = \frac{x}{\sqrt{1-x^2}} dx$ $u = 1-x^2$
 $du = -2x dx$

Sketch y given the initial condition $y(0) = 1$. $-\frac{1}{2} du = x dx$

$$-\frac{1}{2} \int \frac{du}{\sqrt{u}} = -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} = -\sqrt{u}$$

$$\ln y = -\sqrt{1-x^2} + C$$

$$y = ce^{-\sqrt{1-x^2}}$$

$$1 = ce^{-\sqrt{1-0^2}} \quad c = e$$

$$y = e e^{-\sqrt{1-x^2}}$$

$$y = e^{1-\sqrt{1-x^2}}$$

$$\frac{5-22}{33-36}$$
$$38$$

Example 4

Solve $y' = x \sec y$

Sketch y given the initial condition $y(0) = 1$.

Example 5

A cylindrical tank of height 9 feet and radius 2 feet is filled with water. Water drains through a square hole of side 1 inch in the bottom. Determine the water level $y(t)$ at time t (seconds). How long does it take for the tank to go from full to empty?

