9.4 examples

Thursday, February 28, 2019 10:01 AM

9.4 examples

Calculus BC – section 9.4 – The Logistic Equation

$$\frac{dy}{dt} = ky\left(1-\frac{y}{A}\right) \qquad Y = \frac{A}{1-e^{-kt}/C} = \frac{A}{1-Ce^{-kt}}$$

Example 1

Solve $\frac{dy}{dt} = 0.3y(4-y)$ with initial condition y(0) = 1

$$\frac{dy}{dt} = 0.3y \cdot 4\left(1-\frac{y}{4}\right) = \overset{k}{1.2}y\left(1-\frac{y}{\underset{A}{4}}\right)$$

$$Y = \frac{4}{1-ce^{-1.2t}} \qquad Y = \frac{4}{1+3e^{-1.2t}}$$

S42: 1-4

Example 2

A 10,000-acre forest has a carrying capacity of 1000 deer. Assume that the deer population grows logistically with growth constant $k = 0.4\ yr^{-1}$.

a) Find the deer population P(t) if the initial population P_0 is 100.

$$P = \frac{1000}{1 - Ce^{-kt}} \qquad 100 = \frac{1000}{1-C} \qquad C = -9$$

$$P = \frac{1000}{1 + 9e^{-0.4t}}$$

b) How long does it take the deer population to reach 500?

$$500 = \frac{1000}{1 + 9e^{-0.4t}}$$

$$500 + 4500e^{-0.4t} = 1000$$

$$e^{-0.4t} = \frac{1}{9}$$

$$-0.4t = \ln 1/9$$

$t = 5.493$ years

A rumor spreads through a school with 1000 students. The number of students grows logistically. 5 students know it initially. After 1 hour 100 students know it. When do 500 students know it?

$$P = \frac{1000}{1 - Ce^{-kt}}$$

$$5 = \frac{1000}{1 - C} \qquad C = -199$$

$$P = \frac{1000}{1 + 199e^{-kt}}$$

$$100 = \frac{1000}{1 + 199e^{-k}}$$

$$100 + 19{,}900e^{-k} = 1000$$

$$19{,}900e^{-k} = 900$$

$$e^{-k} = 0.0452$$

$$-k = -3.096$$

$$k = 3.096$$

$$500 = \frac{1000}{1 + 199e^{-3.096t}}$$

$$1 + 199e^{-3.096t} = 2$$

$$199e^{-3.096t} = 1$$

$$e^{-3.096t} = \frac{1}{199}$$

$$-3.096t = -5.293$$

$$t = 1.709$$

$$\underline{542}$$

5, 7, 9