Calculus Study Guide: 10.2

Calculate S_1, S_2 and S_3 and then find the sum S of the telescoping series $S = \sum_1^{\infty} \frac{1}{4n^2-1}$, using the identity $\frac{1}{4n^2-1} = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

$\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots$

$\frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots\right)$

$S_{2N} = \frac{1}{2}\left(1 - \frac{1}{2N+1}\right)$

$\lim_{N \to \infty} S_{2N} = S = \frac{1}{2}$

S_1 : $\frac{1}{3}$

S_2 : $\frac{1}{3} + \frac{1}{15}$

S_3 : $\frac{1}{3} + \frac{1}{15} + \frac{1}{35}$

S : ________

sin[π / 4] + sin[2π / 6] + sin[3π / 8] + sin[4π / 10] + ...

$\sin\left(\frac{n\pi}{2+2n}\right) = a_n$

$a_n \to \sin\frac{\pi}{2} = 1$

___converges. sum = ________ ✓___diverges Divergence Test

test used: ____

7/8 – 49/64 + 343/512 – 2,401/4,096 + ... geometric

$r = -7/8$

$\frac{7/8}{1 - -7/8} = \frac{7/8}{15/8} = \frac{7}{15}$

✓___converges. sum = $\frac{7}{15}$ ___diverges

test used: geometric series

$\sum_{n=1}^{\infty} \frac{7+3^n}{5^n} = \sum \frac{7}{5^n} + \sum \frac{3^n}{5^n} = \left(\frac{7}{5} + \frac{7}{25} + \frac{7}{125} + \ldots\right) + \left(\frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \ldots\right)$

$\frac{\frac{7}{5}}{1 - \frac{1}{5}} = \frac{7}{4}$ $\frac{\frac{3}{5}}{1 - \frac{3}{5}} = \frac{3}{2}$

✓___converges. sum = 3.25 ___diverges

test used: ____

Formula: $s = 0.5gt^2$, where $g = 32\ ft/s^2$, describes how far a dropped object, under the influence of gravity, travels as a function of time.

A superball is dropped from the top of a 200-foot building. On each bounce, it returns to 65% of its height on the previous bounce.

What is the upper limit on how far the ball might travel before it is done bouncing?

$$200 + 130(2) + 84.5(2) + \cdots$$

$$200 + \frac{260}{1-0.65} = 942.857$$

Answer: 942.857'

What is the upper limit on how long the ball might bounce before it is done?

$$t = \frac{\sqrt{s}}{4}$$

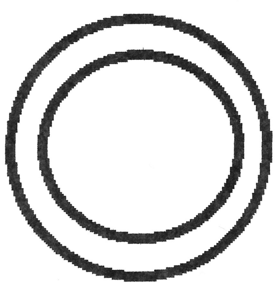
$$\frac{\sqrt{200}}{4} + 2\frac{\sqrt{130}}{4} + 2\frac{\sqrt{84.5}}{4} + \cdots$$

$$\frac{\sqrt{200}}{4} + \frac{\frac{2\sqrt{130}}{4}}{1-\sqrt{0.65}} = 32.955$$

Answer: 32.955 s

The outermost circle below has radius = 10. The inner concentric circle has radius = 8. Assume that the pattern continues, and there is an infinite number of concentric circles, each one with a radius 80% as long as the radius of the next larger circle. What is the sum of the areas of all of the circles?

$$100\pi + 64\pi + 40.96\pi + \cdots$$



$$\frac{100\pi}{1-0.64} = 872.664$$

answer: 872.664