Calculus Study Guide: 10.3

Indicate whether the series converges or diverges, and state which test you used.

$1 + 1/2 + 1/3 + 1/4 + 1/5 + \ldots$ Harmonic Series or use Integral Test with $\int_1^\infty \frac{1}{x}\,dx$

___converges. sum = _____ ✓ diverges

test used: _____

Indicate whether or not each series converges. State the test you used.

$\sum_{n=1}^{\infty} \frac{4n^3}{e^{n^4}}$ Integral Test

$4\int_1^\infty \frac{x^3}{e^{x^4}}\,dx$

$u = x^4 \quad du = 4x^3\,dx$

$\int \frac{1}{e^u}\,du \rightarrow \int e^{-u}\,du$

$= -e^{-u}\Big|_1^\infty$

$= -\left(\frac{1}{e^\infty} - \frac{1}{e^1}\right) = \frac{1}{e}$ converges

✓ converges ___diverges

test used: Integral Test

$\sum_{k=0}^{\infty} \frac{1}{k^{\left(\frac{29}{30}\right)}}$

___converges ✓ diverges

test used: $\frac{1}{n^p}$ rule

$$\sum_{k=0}^{\infty} \frac{1}{k^{\left(\frac{30}{29}\right)}}$$

✓ converges ___ diverges

test used: p rule

$$\sum_{k=1}^{\infty} \frac{5}{3^k - 1}$$

$$\frac{\frac{5}{3^k}}{\frac{5}{3^k - 1}} = \frac{(3^k - 1) \cdot 5}{3^k \cdot 5}$$

$$\frac{3^k - 1}{3^k} \to 1$$

$\sum \frac{5}{3^k}$ converges (geometric series)

✓ converges ___ diverges test used: Limit Comparison Test

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{8n^2 - 3n}} > \sum \frac{1}{2n^{2/3}}$$

← divergent series ($2/3 < 1$)

___ converges ✓ diverges test used: Comparison Test

$\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$ $\quad \frac{\sin^2 n}{n^2} \leq \frac{1}{n^2}$ ← converges by $\frac{1}{n^p}$ rule

✓ converges ___ diverges test used: Comparison Test

$\sum_{n=2}^{\infty} \frac{1}{n^3 - \sqrt[3]{n}}$ $\quad \frac{\frac{1}{n^3}}{\frac{1}{n^3 - \sqrt[3]{n}}} = \frac{n^3 - \sqrt[3]{n}}{n^3} = 1 - \frac{1}{n^{8/3}} \to 1$

✓ converges ___ diverges test used: Limit Comparison Test

$\sum_{n=1}^{\infty} \left(\frac{n}{3n+12}\right)^n$ $\quad < \left(\frac{1}{3}\right)^n$ ← convergent geometric series

✓ converges ___ diverges

test used: Comparison Test