Calculus Study Guide: 10.4

Classify each series as absolutely convergent, conditionally convergent or divergent.

$\sum_{k=3}^{\infty} \frac{(-1)^k \ln k}{k}$ converges by Leibniz Test $a_n \to 0$

$\lim_{k\to\infty} \frac{\ln k}{k}$ is indeterminate. $\xrightarrow[\text{Rule}]{\text{L'Hopital's}} \frac{\frac{1}{k}}{1} \to 0$

but $\sum \frac{\ln k}{k}$ diverges. $\frac{\ln k}{k} > \frac{1}{k}$

diverges by Comparison Test, $\frac{1}{n^p}$ Rule

___ converges absolutely ✓ converges conditionally ___ diverges

$\sum_{n=1}^{\infty} \frac{(-4)^n}{n^2}$ fails the Leibniz Test. $a_n \not\to 0$

$\lim_{n\to\infty} \frac{4^n}{n^2}$ is indeterminate. $\xrightarrow[\text{Rule}]{\text{L'Hopital's}} \frac{\ln 4 \cdot 4^n}{2n}$

$\xrightarrow[\text{Rule}]{\text{L'Hopital's}} \frac{(\ln 4)^2 4^n}{2} \to \infty$

___ converges absolutely ___ converges conditionally ✓ diverges

$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} = \cos \pi + \frac{\cos(2\pi)}{2} + \frac{\cos(3\pi)}{3} + \frac{\cos(4\pi)}{4}$

$= -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$

Alternating Harmonic Series

___ converges absolutely ✓ converges conditionally ___ diverges

For the stated value of n, find the largest error that results if the sum of the series is approximated by the nth partial sum.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ n = 5

first omitted term $= \frac{1}{6!}$

error $\le \frac{1}{6!}$

Find the smallest number of terms for which S_N (the nth partial sum) approximates the sum of the series to the stated accuracy.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ |error| ≤ 0.005

$\frac{1}{\sqrt{n+1}} < 0.005$

$\sqrt{n+1} > 200$

$n+1 > 40,000$

$n > 39,999$

Include 39,999 terms.

First omitted term $= \frac{1}{\sqrt{40,000}} = \frac{1}{200}$

error $\le \frac{1}{200}$