Calculus Study Guide: 10.5

Determine whether the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{k}{2^k}$$

$\sqrt[k]{k} \to 1$

Root Test

$\sqrt[k]{\frac{k}{2^k}} = \frac{1}{2} < 1$

✓ converges ___ diverges

$$\sum_{k=1}^{\infty} \frac{k^k}{k!}$$

Ratio Test

$\dfrac{\frac{(k+1)^{k+1}}{(k+1)!}}{\frac{k^k}{k!}} \to \dfrac{(k+1)^{k+1} \cdot k!}{(k+1)!\, k^k}$

$\dfrac{(k+1)^{k+1}}{(k+1) \cdot k^k}$

$\dfrac{(k+1)^k}{k^k} \to \left(1 + \frac{1}{k}\right)^k$

$\to e > 1$

diverges

___ converges ✓ diverges

$$\sum_{k=1}^{\infty} \frac{k!}{k^3}$$

Ratio Test

$\dfrac{\frac{(k+1)!}{(k+1)^3}}{\frac{k!}{k^3}}$

$\dfrac{(k+1)! \cdot k^3}{k!\,(k+1)^3}$

$\dfrac{(k+1) \cdot k^3}{(k+1)^3}$

$(k+1) \cdot 1 \to \infty$

___ converges ✓ diverges

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2k-1} + \cdots$$

Integral Test

$\int_1^\infty \frac{dx}{2x-1} = \lim_{R \to \infty} \frac{1}{2}\ln|2x-1| \Big|_1^R = \lim_{R\to\infty} \frac{1}{2}\ln(2R-1) - \frac{1}{2}\ln 1 \to \infty$

___ converges ✓ diverges

$$\sum_{k=1}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$$

Root Test

$$\sqrt[k]{\left(\frac{4k-5}{2k+1}\right)^k} \to \frac{4k-5}{2k+1} \to 2 > 1$$

___ converges ✓ diverges

$$\sum_{k=1}^{\infty}\frac{1}{(\ln(k+1))^k}$$

Root Test

$$\sqrt[k]{\frac{1}{(\ln(k+1))^k}} \to \frac{1}{\ln(k+1)} \to 0$$

✓ converges ___ diverges

$$1+\frac{1\cdot 2}{1\cdot 3}+\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5}+\frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7}+\cdots$$

written recursively, Use Ratio test

$a_1 = 1 \quad a_{n+1} = a_n\left(\frac{n}{2n-1}\right)$ $\quad \frac{a_{n+1}}{a_n} \to \frac{1}{2} < 1$

or $\frac{a_{n+1}}{a_n} = \frac{n}{2n-1}$

✓ converges ___ diverges

$$\sum_{k=1}^{\infty}\frac{k^2}{5^k}$$

$\sqrt[k]{k^2} \to 1^2 = 1$

$\sqrt[k]{\frac{k^2}{5^k}} = \frac{1}{5} < 1$

Root Test

$\sqrt[k]{k} \to 1$

✓ converges ___ diverges

$$\sum_{k=1}^{\infty}k\left(\frac{2}{3}\right)^k$$

Root Test

$\sqrt[k]{k(2/3)^k} \to 1 \cdot 2/3 = 2/3 < 1$

✓ converges ___ diverges