Calculus Study Guide: 10.6

Find the interval of convergence of the power series. Remember to check endpoints.

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (x+5)^n$$

Root Test

$$\sqrt[n]{\left|\left(\frac{3}{4}\right)^n (x+5)^n\right|} = \left|\left(\frac{3}{4}\right)(x+5)\right|$$

$$\left|\frac{3}{4}(x+5)\right| < 1$$

$$|x+5| < 4/3$$

$$|x - -5| < 4/3$$

x converges on $(-6\tfrac{1}{3}, -3\tfrac{2}{3})$

endpoints: $x = -6\tfrac{1}{3}$

$$\sum \left(\frac{3}{4}\right)^n \left(-\frac{4}{3}\right)^n = -1 + 1 - 1 \cdots$$

diverges (Divergence Test)

$x = -3\tfrac{2}{3}$:

$$\sum \left(\frac{3}{4}\right)^n \left(\frac{4}{3}\right)^n = 1 + 1 + 1 \cdots$$

diverges (Divergence Test)

$$\boxed{(-6\tfrac{1}{3}, -3\tfrac{2}{3})}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{x^{2(n+1)}}{2(n+1)!}}{\frac{x^{2n}}{(2n)!}}$$

$$= \frac{x^{2n+2}\,(2n)!}{x^{2n}\,(2n+2)!}$$

$$\frac{x^2}{(2n+2)(2n+1)} \to 0 \quad \text{(for all } x\text{)}$$

converges for all x

Which function f(x) is equivalent to the power series $\sum_{n=0}^{\infty} \frac{x^n}{3^n}$? $= 1 + \frac{x}{3} + \frac{x^2}{9} + \cdots$

geometric series with $r = \frac{x}{3}$

use $\frac{c}{1-r}$

$$\frac{1}{1 - x/3} = f(x)$$

Expand the function h(x) = $\frac{3}{1+2x}$ in a power series with center c = 0 and determine the set of x for which the expansion is valid.

$$\frac{3}{1+2x} = \frac{3}{1--2x} \qquad c=3 \quad r=-2x$$

$$3 - 6x + 12x^2 - 24x^3 + \cdots$$

$|r| < 1$

$|-2x| < 1$

$|x| < \frac{1}{2}$, so $(-\frac{1}{2}, \frac{1}{2})$ is interval of convergence

Expand the function i(x) = $\frac{1}{1+x}$ in a power series with center c = 0.

$$\frac{1}{1+x} = \frac{c}{1-r} \qquad r=-x \quad c=1$$

$$1 - x + x^2 - x^3 + \cdots$$

$|r| < 1$

$|-x| < 1$ interval of convergence $= (-1, 1)$

Use i(x) and its power series to obtain a power series for p(x) = ln(1 + x).

$$\ln(1+x) = \int \frac{1}{1+x}\,dx$$

So integrate the power series for $i(x)$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

(same interval of convergence)

Use i(x) and its power series to obtain a power series for q(x) = $\frac{-1}{(1+x)^2}$

$$\frac{-1}{(1+x)^2} = \frac{d}{dx}\frac{1}{1+x}$$

So differentiate the power series for $i(x)$

$$\frac{-1}{(1+x)^2} = -1 + 2x - 3x^2 + 4x^3 - \cdots$$

(same interval of convergence)