Find the functions with the following Maclaurin series.

$4x - \frac{4^3x^3}{3!} + \frac{4^5x^5}{5!} - \dots$ $\quad \sin(4x)$

$1 - 4x + 4^2x^2 - 4^3x^3 + \dots$ $\quad \frac{1}{1+4x}$

Find n such that $|T_n(0.8) - \ln 0.8| \leq 10^{-5}$, where T_n is the Taylor polynomial for $f(x) = \ln x$ at $a = 1$.

$f'(x) = \frac{1}{x}$ $\quad f^{(4)}(x) = -6x^{-4}$ $\quad f^{(7)}(x) = 720x^{-7}$

$f''(x) = -x^{-2}$ $\quad f^{(5)}(x) = 24x^{-5}$

$f'''(x) = 2x^{-3}$ $\quad f^{(6)}(x) = -120x^{-6}$

$f^{(n)}(x) = (n-1)!(-1)^{n+1} \cdot x^{-n}$

$f^{(n+1)}(x) = n!(-1)^n x^{-(n+1)}$

Error Bound: $\frac{n!(-1)^n u^{-(n+1)}(x-1)^{n+1}}{(n+1)!} = \frac{u^{-(n+1)}(x-1)^{n+1}}{(n+1)}$

Maximize $u^{-(n+1)} = \frac{1}{u^{n+1}}$ by choosing $u = 0.8$

Error Bound: $\frac{1}{0.8^{n+1}} \frac{(-0.2)^{n+1}}{n+1} = \frac{\left(-\frac{1}{4}\right)^{n+1}}{n+1} \leq 10^{-5}$

$\frac{\left(-\frac{1}{4}\right)^7}{7} \leq 10^{-5}$ Include 6 terms in the polynomial

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Or, it is an alternating series, so use the error bound for an alternating series.

$1(x-1) - \frac{(x-1)^2}{2} + \frac{2(x-1)^3}{3!} - \frac{6(x-1)^4}{4!} + \frac{24(x-1)^5}{5!} - \frac{120(x-1)^6}{6!} + \frac{720(x-1)^7}{7!} - \dots$ 2

The order-7 term $\frac{720(0.2)^7}{7!} < 10^{-5}$ so stop at the x^6 term.