Calculus Study Guide: 11.4

Find the area of the region in the first quadrant within the cardioid r = 1 – cos θ

$$\frac{1}{2}\int_0^{\pi/2} (1-\cos\theta)^2\, d\theta = 0.178$$

Find the area between the outer loop and the inner loop for the limacon r = 2 + (3 cos θ)

θ	r
0	5

When does the curve go through the origin?

$$2 + 3\cos\theta = 0$$

$$\cos\theta = -2/3$$

$$\theta = 2.301$$

$$\frac{1}{2}\cdot 2\left[\int_0^{2.301} (2+3\cos\theta)^2\, d\theta - \int_{2.301}^{\pi} (2+3\cos\theta)^2\, d\theta\right]$$

$$= 25.821$$

Find the slope of r = 1 – cos θ, at θ = π/4.

$$y = r\sin\theta = (1-\cos\theta)\sin\theta$$

$$\frac{dy}{d\theta} = \sin\theta^2 + \cos\theta - \cos^2\theta \Big|_{\pi/4} = \frac{1}{2} + \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}}{2}$$

$$x = r\cos\theta = (1-\cos\theta)\cos\theta$$

$$\frac{dy}{d\theta} = \sin\theta\cos\theta - \sin\theta(1-\cos\theta)$$

$$= \frac{1}{2} - \frac{1}{\sqrt{2}}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \frac{1}{\sqrt{2}} + \frac{1}{2} =$$

$$1 - 1/\sqrt{2}$$

$$\text{slope} = \frac{\sqrt{2}/2}{1 - 1/\sqrt{2}} = 2.414$$

Find the slope of r = 2 + (3 cos θ) at θ = -π/3.

$y = r\sin\theta = (2+3\cos\theta)\sin\theta$

$\frac{dy}{d\theta} = -3\sin^2\theta + \cos\theta(2+3\cos\theta) = -3\sin^2\theta + 2\cos\theta + 3\cos^2\theta$

$= -3\left(-\sqrt{3}/2\right)^2 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 = -3\left(\frac{3}{4}\right) + 1 + \frac{3}{4} = -\frac{1}{2}$

$x = r\cos\theta = (2+3\cos\theta)\cos\theta$

$\frac{dx}{d\theta} = -3\sin\theta\cos\theta + (2+3\cos\theta)(-\sin\theta)$

$= -3\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(2+3\left(\frac{1}{2}\right)\right)\left(\sqrt{3}/2\right) = \frac{3\sqrt{3}}{4} + \sqrt{3} + \frac{3\sqrt{3}}{4}$

$= 4.330$

$\frac{dy}{dx} = \frac{-\frac{1}{2}}{4.330} = -0.115$

Find the path length of r = 1 – cos θ on the interval (0, π/2).

$\frac{dr}{d\theta} = \sin\theta$

$\int_0^{\pi/2} \sqrt{(1-\cos\theta)^2 + \sin^2\theta}\, d\theta$

$= 1.171$

Find the path length of r = 2 + (3 cos θ) on the interval (-π/6, π/6).

$\frac{dr}{d\theta} = -3\sin\theta$

$\int_{-\pi/6}^{\pi/6} \sqrt{(2+3\cos\theta)^2 + 9\sin^2\theta}\, d\theta$

$= 5.178$