

Calculus Study Guide: section 3.10

Find $\frac{dy}{dx}$ if $y = \ln(\csc 3x)$ at $x = \pi/4$

$$\frac{dy}{dx} = \frac{1}{\csc 3x} \cdot -\csc 3x \cot 3x \cdot 3 = -3 \cot 3x$$

$$\rightarrow -3 \cot 3\pi/4 = 3$$

$$\frac{dy}{dx} = \underline{\hspace{10em}}$$

Given $y = \log_4 x^2$, evaluate $y'(3)$

$$\log_4 x^2 = \frac{\ln x^2}{\ln 4}$$

$$\frac{d}{dx} \left(\frac{\ln x^2}{\ln 4} \right) = \frac{1}{\ln 4} \frac{d}{dx} (\ln(x^2))$$

$$\frac{1}{\ln 4} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{x \ln 4}$$

$$\frac{2}{3 \ln 4}$$

answer: $\underline{\hspace{10em}} \frac{2}{3 \ln 4}$

Find the derivative $\frac{dy}{dx}$.

$$y = 4^x$$

$$\frac{dy}{dx} = \underline{\hspace{10em}} 4^x \ln 4$$

$$y = x^2 4^{3x}$$

$$\frac{dy}{dx} = 2x 4^{3x} + x^2 \cdot 4^{3x} \cdot (\ln 4 \cdot 3)$$

$$\frac{dy}{dx} = \underline{\hspace{10em}}$$

Find an equation of the tangent line at the point indicated.

$$y = \pi^{2x-5}, x = 2$$

$$\frac{dy}{dx} = \pi^{2x-5} \cdot (\ln \pi) \cdot 2$$

$$= \pi^{-1} \cdot \ln \pi \cdot 2 = \frac{2(\ln \pi)}{\pi}$$

answer: _____

Given $y = (\sec x)^x$, find $y'(\frac{\pi}{3})$

$$\ln y = \ln (\sec x)^x = x \ln (\sec x)$$

$$\frac{d}{dx} (\ln y = x \ln (\sec x))$$

$$= \frac{1}{y} \frac{dy}{dx} = \ln (\sec x) + \frac{1}{\sec x} \cdot \sec x \tan x \cdot x$$

$$\frac{dy}{dx} = y (\ln (\sec x) + x \tan x)$$

$$= (\sec x)^x (\ln (\sec x) + x \tan x)$$

answer: _____