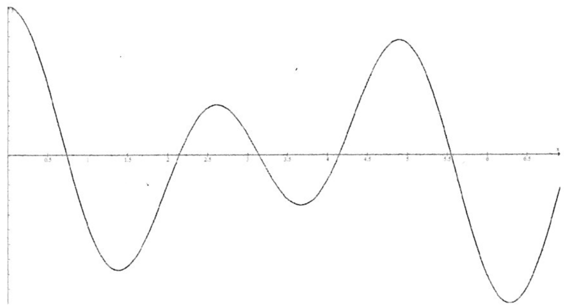
Calculus BC Study Guide: 3.4 – 3.8

Name:

Shown below are f(x), $f'(x)$ and $f''(x)$. Label each graph.

f'



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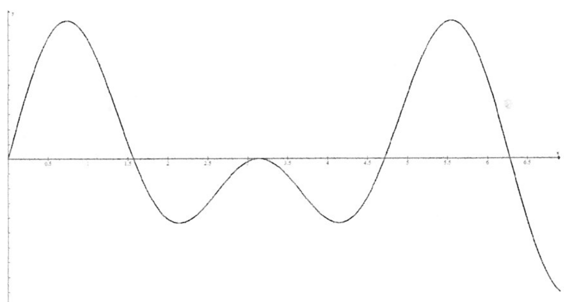

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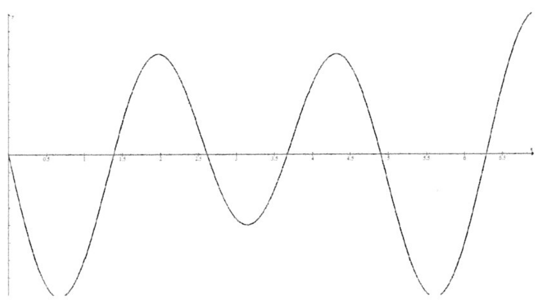


Galileo's equation for the position s(t) of a freely-falling object (neglecting air resistance):

$s(t) = s_0 + v_0 t - 0.5gt^2$

A snowball is thrown from the top of a 100-foot-tall building. The snowball is thrown upward with a velocity of 32 ft/s at time t = 0. (Use g = 32 ft/s²)

a) What is the maximum height above the ground, in feet, that the snowball reaches?

A) 116 B) 124 C) 134 D) 108 E) 140

$s = -16t^2 + 32t + 100$

$v = -32t + 32 = 0$

$t = 1$

$s(1) = -16 + 32 + 100 = 116$

b) After how many seconds is the height of the snowball 100 ft, when it is on its way down?

A) 1.0 B) 3.0 C) 2.5 D) 2.0 E) 1.5

c) Assume that the snowfall falls for another 1.692 seconds, after its position given in part (b). How would you calculate the snowball's velocity when it hits the ground?

A) 32 – 32(1.692) B) -32 – 32(1.692) C) 100 - 32(1.692) - 16(1.692)^2

D) 100 + 32(1.692) - 16(1.692)^2 E) -32

The population of East Bumblebee in recent years is given in the table below.
The last row of the table shows the change in population from the previous year.

Population (P)					
2006	2007	2008	2009	2010	2011
10,000	10,100	10,400	10,900	11,600	12,500
	100	300	500	700	900

Is $\frac{dP}{dt} > 0$, $\frac{dP}{dt} = 0$, or $\frac{dP}{dt} < 0$? (Circle one.)

Is $\frac{d^2P}{dt^2} > 0$, $\frac{d^2P}{dt^2} = 0$, or $\frac{d^2P}{dt^2} < 0$? (Circle one.)

Find the acceleration function of a helicopter whose height is h(t) = 2t^3 – 0.1e^t + t + 200.

$v = 6t^2 - 0.1e^t + 1$

a(t) = $12t - 0.1e^t$

Find its acceleration at time t = 5.

a(5) = $60 - 0.1e^5 = 45.158$ ____________________

Is the helicopter speeding up or slowing down from t = 0 to t = 5?

$v(5) = 136.158$

$v > 0$

Speeding up   Slowing down $a > 0$

Find the equation of the tangent line to f(x) = sin x tan x at x = π/6.

$f(\pi/6) = \sin \pi/6 \tan \pi/6 = \frac{1}{2} \cdot \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$

$f'(x) = \cos x \tan x + \sin x \sec^2 x$ $\quad f'(\pi/6) = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} + \frac{1}{2} \cdot \frac{4}{3}$

$= \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

$y - \frac{1}{2\sqrt{3}} = \left(\frac{7}{6} \right)(x - \pi/6)$

answer: ____________________

Find the derivative:

$y = \sqrt{\sec 2x} = (\sec 2x)^{1/2}$

$y' = \frac{1}{2} (\sec 2x)^{-1/2} (\sec 2x \tan 2x) \cdot 2$

(optional) $\frac{\sec 2x \tan 2x}{\sqrt{\sec 2x}} = \sqrt{\sec 2x} \tan 2x$

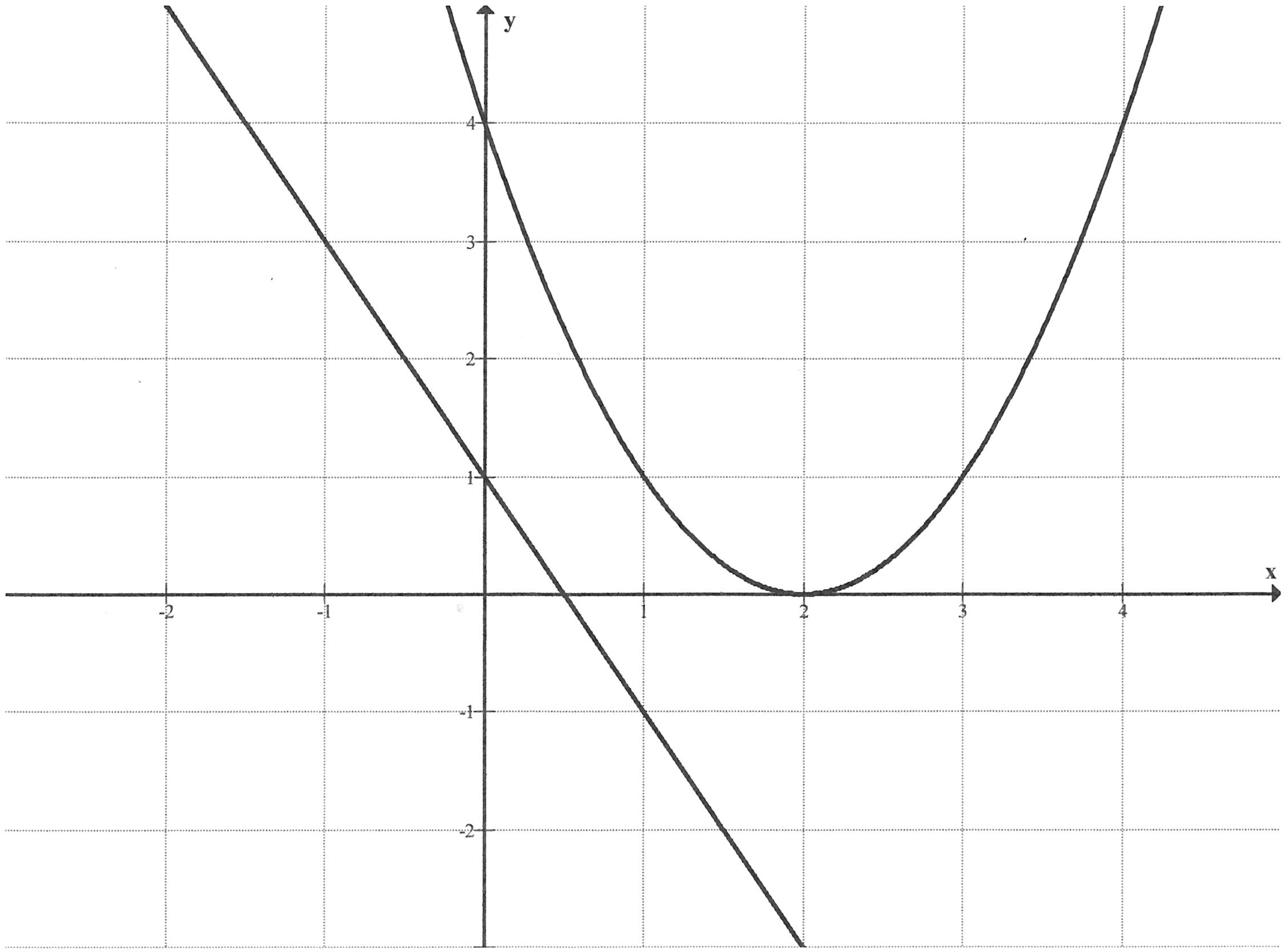
$\frac{dy}{dx} =$ ____________________

$y = (1 + csc^3 2x)^5$

$5(1 + \csc^3 2x)^4 \cdot (3 \csc^2 2x)(-\csc 2x \cot 2x) \cdot 2$

phew

$\frac{dy}{dx} =$ ____________________



In the graph above, f(x) is the straight line and g(x) is the curved function.
Given the function H(x) = f(g(x)), estimate $H'(3)$.
This can be done graphically; you don't need to figure out what functions f(x) and g(x) are.

$$H'(x) = f'(g(x)) \cdot g'(x) = f'(g(3)) \cdot g'(3)$$
$$= f'(1) \cdot 2$$
$$= -4$$

$H'(3) \approx$ ____________

According to the U.S. standard atmospheric model, developed by the National Oceanic and Atmospheric Administration for use in aircraft and rocket design, atmospheric temperature T (in degrees Celsius), pressure P (kPa = 1,000 Pascals), and altitude h (meters) are related by the formulas (valid in the troposphere h ≤ 11,000):

$T = 15.04 - 0.000649h,$ $\qquad$ $P = 101.29 + \left(\dfrac{T+273.1}{288.08}\right)^{5.256}$

Calculate $\dfrac{dP}{dh}$. Then estimate the change in P (in Pascals, Pa) per additional meter of altitude when h = 3,000.

$$\frac{dP}{dh} = \frac{dP}{dT}\cdot\frac{dT}{dh} \qquad \frac{dP}{dT} = 5.256\left(\frac{T+273.1}{288.08}\right)^{4.256}\cdot\frac{1}{288.08}$$

$$T = 15.04 - 0.000649(3000) = 13.093$$

$$\frac{dP}{dT} = 5.256\left(\frac{13.093+273.1}{288.08}\right)^{4.256}\cdot\frac{1}{288.08} = 0.1089$$

$$\frac{dT}{dh} = -0.000649 \qquad \frac{dP}{dT}\cdot\frac{dT}{dh} = -0.0000707$$

−0.0000707 Pa per meter altitude

Given the relation: $\sin(x + y) + x^2 = 1$, estimate the slope of the curve at the point (1, 2.15).

$$\frac{d}{dx}\left(\sin(x+y) + x^2 = 1\right) = \cos(x+y)(1+y') + 2x = 0$$

$$\cos(3.15)(1+y') + 2 = 0 \qquad 1+y' = \frac{-2}{\cos(3.15)}$$

$$y' = 1$$

Slope ≈__________________________