Study Guide: 4.1-4.3

1) Find the function's critical point(s) and use the First Derivative Test to classify them as a max or min: f(x) = x ln(x) $f(x) = 1 \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1} = \frac{1}{e}$

First Derivative Test:

$\ln \frac{1}{4} + 1 < 0$ $\ln 1 + 1 > 0$ $\frac{1}{e}$ is a MIN

2) Find the extreme values on the interval:

g(x) = $x^{2/3} - 2x^{1/3}$ on [-1, 3]

$g(-1) = (-1)^{2/3} - 2(-1)^{1/3} = 1 + 2 = 3$

$g(3) = 3^{2/3} - 2(3)^{1/3} = -0.804$

$g'(x) = \frac{2}{3}x^{-1/3} - \frac{2}{3}x^{-2/3} = 0$

$\frac{2}{3\sqrt[3]{x}} - \frac{2}{3\sqrt[3]{x^2}} = 0$

Critical point at $x = 1$

$g(1) = -1$ MIN

g' DNE at $x = 0$

$g(0) = 0$

MAX at $x = -1$

3) Find the critical points and the intervals on which the function is increasing or decreasing, and apply the First Derivative Test to analyze each critical point.

$y = \frac{1}{x^2 + 1}$ $y' = \frac{(x^2+1) \cdot 0 - 1(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2} = 0$

critical point at $x = 0$

$y'(-1) > 1$

$y'(1) < 0$

$x = 0$ is a MAX

y is increasing for $x < 0$

y is decreasing for $x > 0$