

Calculus Study Guide: section 4.1

1) Given  $f(x) = \sec x$ , find its derivative  $f'(x)$ .

$$f'(x) = \sec x \tan x$$

$$f(\pi/4) = \sqrt{2}$$

Find the linearization for  $f(x)$  at  $x = \pi/4$ .

$$f'(\pi/4) = \sqrt{2} \cdot 1 = \sqrt{2}$$

$$y - \sqrt{2} = \sqrt{2}(x - \pi/4)$$

$$L(x) = \sqrt{2}(x - \pi/4) + \sqrt{2}$$

Use the linearization to estimate  $\sec(\pi/4 + 0.1)$

$$L(x) = \sqrt{2}(\pi/4 + 0.1 - \pi/4) + \sqrt{2} = \sqrt{2}(0.1) + \sqrt{2} = 1.1\sqrt{2}$$

$$\sec(\pi/4 + 0.1) \approx \underline{1.1\sqrt{2} = 1.555}$$

What is the % error of this estimate?

$$\text{exact} = 1.579$$

$$\frac{1.579 - 1.555}{1.579} \cdot 100 = 1.531\%$$

2) Use a linearization to estimate  $\sqrt{80}$ .

$$\text{Use } y = \sqrt{x} = x^{1/2} \quad y(81) = 9$$

$$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \quad y'(81) = \frac{1}{2\sqrt{81}} = \frac{1}{18}$$

$$y - 9 = \frac{1}{18}(x - 81) \quad L(x) = \frac{1}{18}(x - 81) + 9 \quad L(80) = \frac{1}{18}(-1) + 9$$

$$= 8 \frac{17}{18} = 8.944$$

3) Use a linearization to estimate  $\frac{1}{\sqrt[3]{26}}$

$$\text{Use } y = x^{-1/3} = \frac{1}{\sqrt[3]{x}}$$

$$y(27) = \frac{1}{\sqrt[3]{27}} = \frac{1}{3} \quad y' = -\frac{1}{3}x^{-4/3} = -\frac{1}{3\sqrt[3]{x^4}}$$

$$= -\frac{1}{3\sqrt[3]{27^4}} = -\frac{1}{3 \cdot 27 \cdot \sqrt[3]{27}} = -\frac{1}{243} \quad y - \frac{1}{3} = -\frac{1}{243}(x - 27)$$

$$L(x) = -\frac{1}{243}(x - 27) + \frac{1}{3} \quad L(26) = -\frac{1}{243}(-1) + \frac{1}{3} = 0.337 \frac{1}{3}$$