Calculus Study Guide: section 4.2

1) Find the critical points of $g(\theta) = \sin^2\theta - \cos\theta$ on the interval $[0, 2\pi]$.

$g'(\theta) = 2\sin\theta\cos\theta + \sin\theta = 0 \qquad \sin\theta(2\cos\theta + 1) = 0$

$\sin\theta = 0 \rightarrow \theta = 0, \pi, 2\pi$ are critical points.

$2\cos\theta + 1 = 0 \rightarrow \cos\theta = -1/2 \rightarrow \theta = 2\pi/3$ is critical point.

$\rightarrow \theta = 4\pi/3$ is critical point.

Find the extreme values (absolute max and min) of $g(\theta)$.

$g(0) = 0 - 1 = -1 \qquad g(\pi) = 0 - -1 = 1 \qquad g(2\pi) = 0 - 1 = -1$

$g(2\pi/3) = \sin^2 2\pi/3 - \cos 2\pi/3 = \frac{3}{4} - -\frac{1}{2} = 5/4$

max at $(2\pi/3, 5/4)$ min at $(0, -1)$ and $(2\pi, -1)$

$g(4\pi/3) = \sin^2 4\pi/3 - \cos 4\pi/3 = \frac{3}{4} - -\frac{1}{2} = 5/4$ (also a max)

2) Find the critical points of $g(x) = x^{2/3} - 2x^{1/3}$ on the interval $[-1, 3]$.

$g'(x) = \frac{2}{3}x^{-1/3} - \frac{2}{3}x^{-2/3} = 0 \qquad \frac{2}{3\sqrt[3]{x}} - \frac{2}{3\sqrt[3]{x^2}} = 0$

critical point at $x = 0$ (g' DNE) Also: $\frac{2}{3\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{x^2}}$

$\sqrt[3]{x} = \sqrt[3]{x^2} \qquad x = x^2$ at $x = 1$

$x = 1$ a second critical point

Find the extreme values of $g(x)$.

$g(-1) = 3 \qquad g(3) = -0.804 \qquad g(0) = 0 \qquad g(1) = -1$

MAX at $(-1, 3)$

MIN at $(1, -1)$

3) Find the critical points of $h(x) = x^{-1/5}$

$h'(x) = -\frac{1}{5}x^{-6/5} = -\frac{1}{5\sqrt[5]{x^6}} = 0$

$x = 0$ is critical pt ($f'(0)$ DNE)

Find the extreme values of $h(x)$ on the interval $[1, 32]$.

$h(1) = 1 \qquad h(32) = \frac{1}{\sqrt[5]{32}} = \frac{1}{2}$

$(1, 1)$ is MAX $\qquad (2, \frac{1}{2})$ is MIN