

Calculus Study Guide: section 4.3

1) Verify the Mean Value Theorem for  $f(x) = \ln x$  on  $[1, 4]$ .

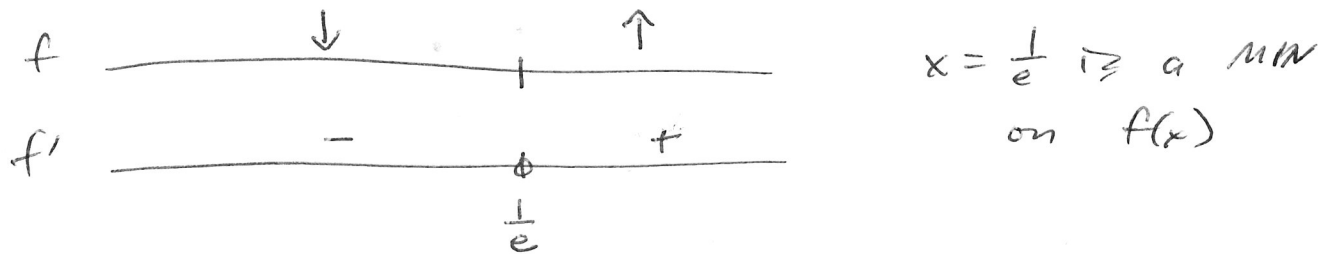
$$\text{average rate of change} = \frac{\ln 4 - \ln 1}{4 - 1} = 0.462$$

$$f'(x) = \frac{1}{x} = 0.462 \quad x = 2.164$$

2) Find the function's critical point(s) and use the First Derivative Test to classify them as a max or min:  $f(x) = x \ln(x)$

$$f'(x) = 1 \cdot \ln x + \frac{x}{x} = \ln x + 1$$

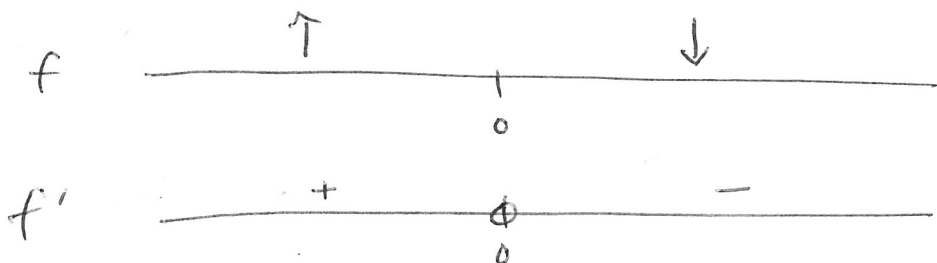
$$\ln x + 1 = 0 \quad \ln x = -1 \quad e^{-1} = x \text{ is a critical point}$$



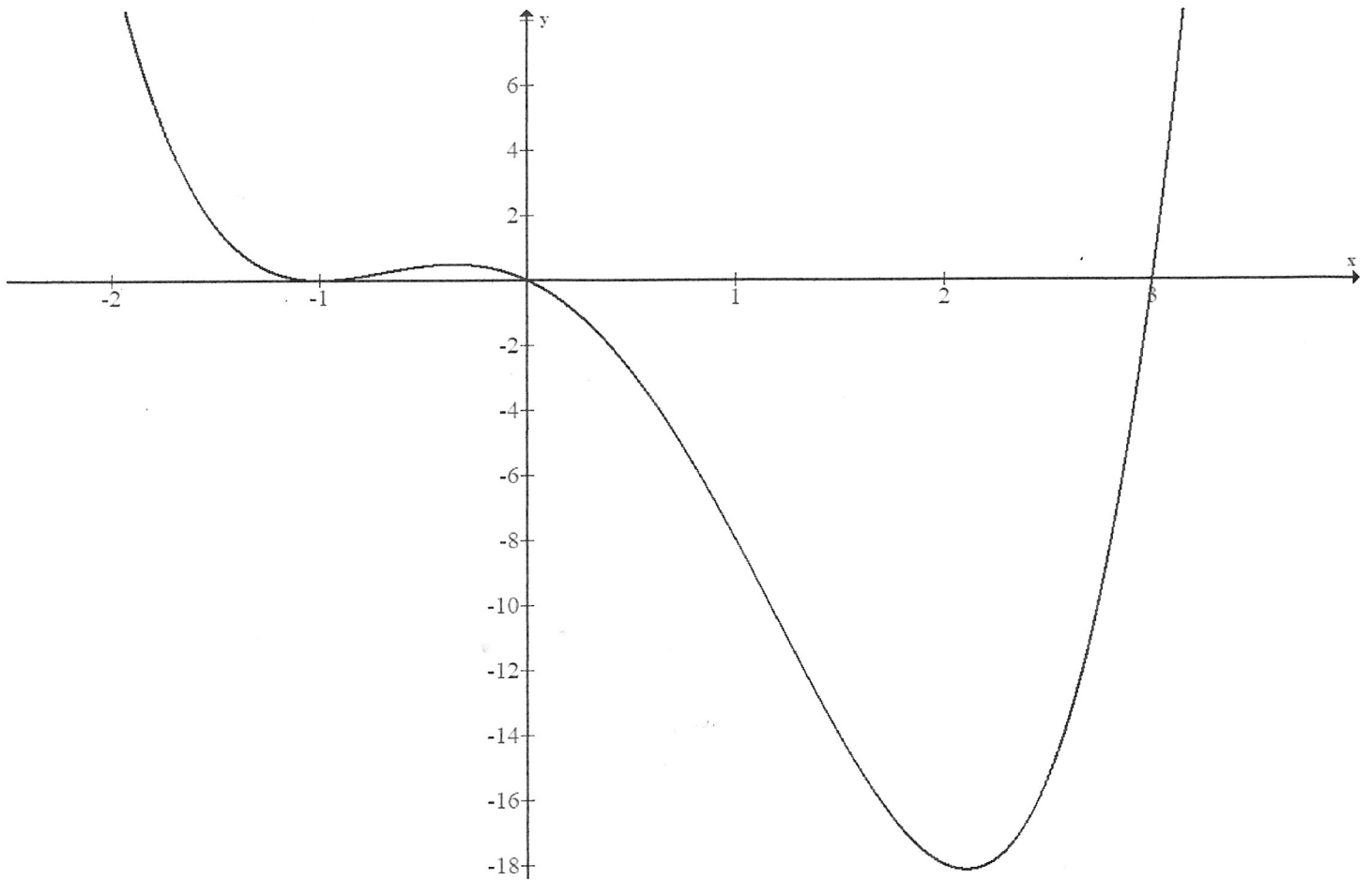
3) Find the critical points and the intervals on which the function is increasing or decreasing, and apply the First Derivative Test to analyze each critical point.

$$y = \frac{1}{x^2 + 1} \quad \frac{dy}{dx} = \frac{(x^2 + 1)(0) - 1(2x)}{(x^2 + 1)^2} = -\frac{2x}{(x^2 + 1)^2}$$

$$\frac{-2x}{(x^2 + 1)^2} = 0 \quad \text{critical point at } x = 0$$



$x = 0$  is a MAX on  $y$



4) Assume that the graph above is of the **derivative** function  $f'(x)$ .

With regards to the original function  $f(x)$ :

On which x-interval(s) is  $f(x)$  increasing?

$$(-2, -1) \cup (-1, 0) \cup (3, 4)$$

On which interval(s) is  $f(x)$  decreasing?

$$(0, 3)$$

At which x-value(s) does  $f$  have a max?

$$x = 0$$

At which x-value(s) does  $f$  have a min?

$$x = 3$$