Calculus Study Guide: 4.4 (Second Derivative Test, concavity)

Draw a curve y = f(x) for which $f'(x)$ and $f''(x)$ have signs as indicated:

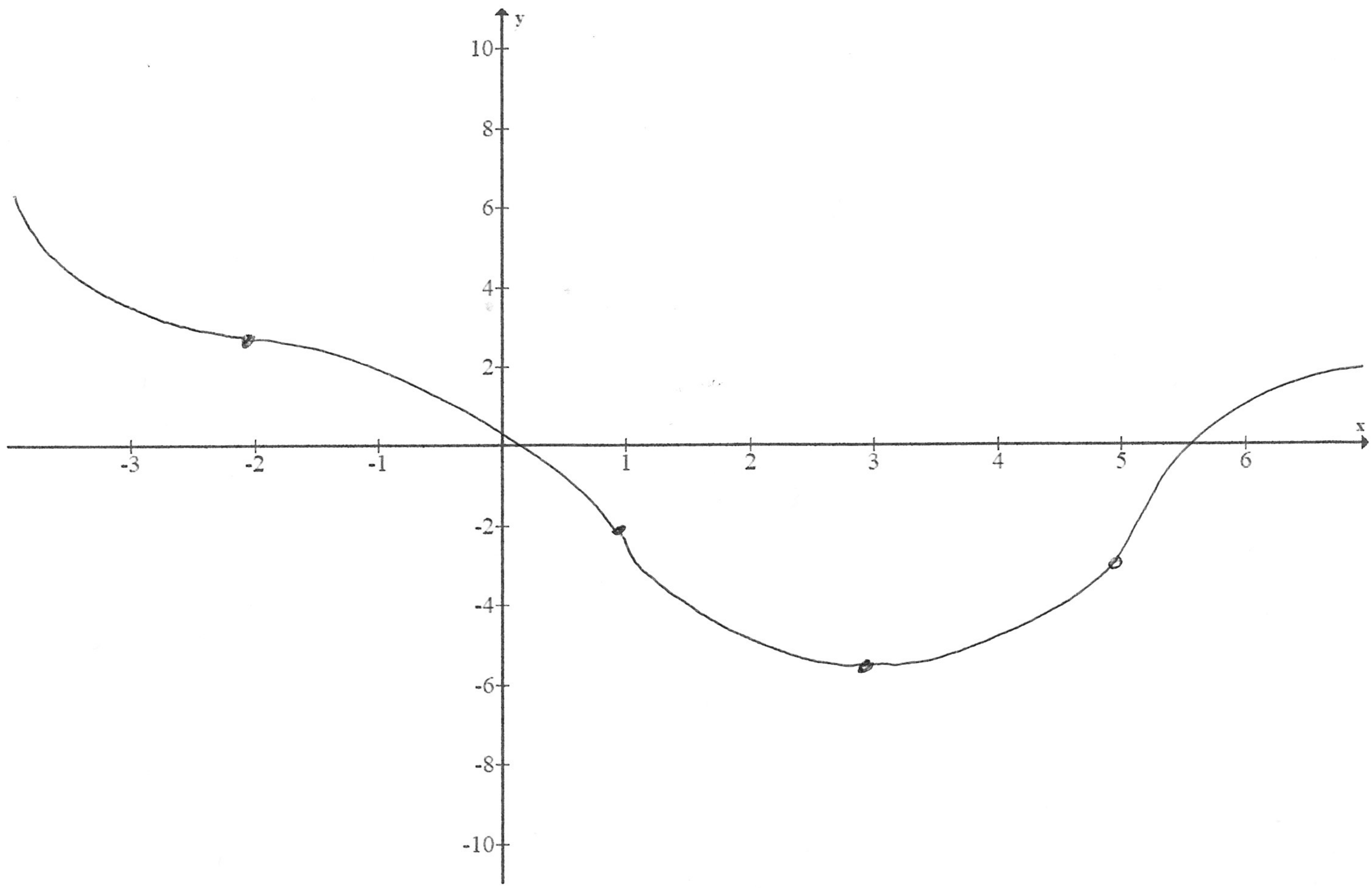
x < -2 - +

(-2, 1) - -

(1, 3) - +

(3, 5) + +

>5 + -



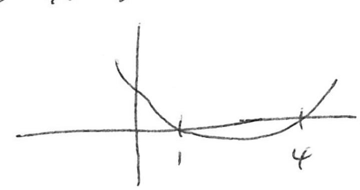
Find the points of inflection:

h(x) = (x² – x)e⁻ˣ

$$h'(x) = (2x-1)e^{-x} - e^{-x}(x^2-x)$$
$$= e^{-x}(-x^2+3x-1)$$
$$h''(x) = -e^{-x}(-x^2+3x-1) + e^{-x}(-2x+3)$$
$$= e^{-x}(x^2-5x+4)$$

$e^{-x} > 0$ $\quad x^2-5x+4=0 \quad (x-4)(x-1)=0$

$x=1,\ x=4$



Find the intervals on which f is concave up or down, the points of inflection, the critical points, and use the Second Derivative Test to classify the critical points.
$f(x) = x^2 - x^{0.5}$

$f'(x) = 2x - \frac{1}{2}x^{-1/2} = 0$ $\quad 2x - \frac{1}{2\sqrt{x}} = 0$ $\quad x = 0$ is an endpoint

critical points:

$2x = \frac{1}{2\sqrt{x}}$ $\quad x^{3/2} = \frac{1}{4}$ $\quad x = \left(\frac{1}{4}\right)^{2/3} = 0.397$

$f''(x) = 2 + \frac{1}{4}x^{-3/2} = 0$ $\quad 2 = -\frac{1}{4x^{3/2}}$ $\quad x^{3/2} = -\frac{1}{8}$

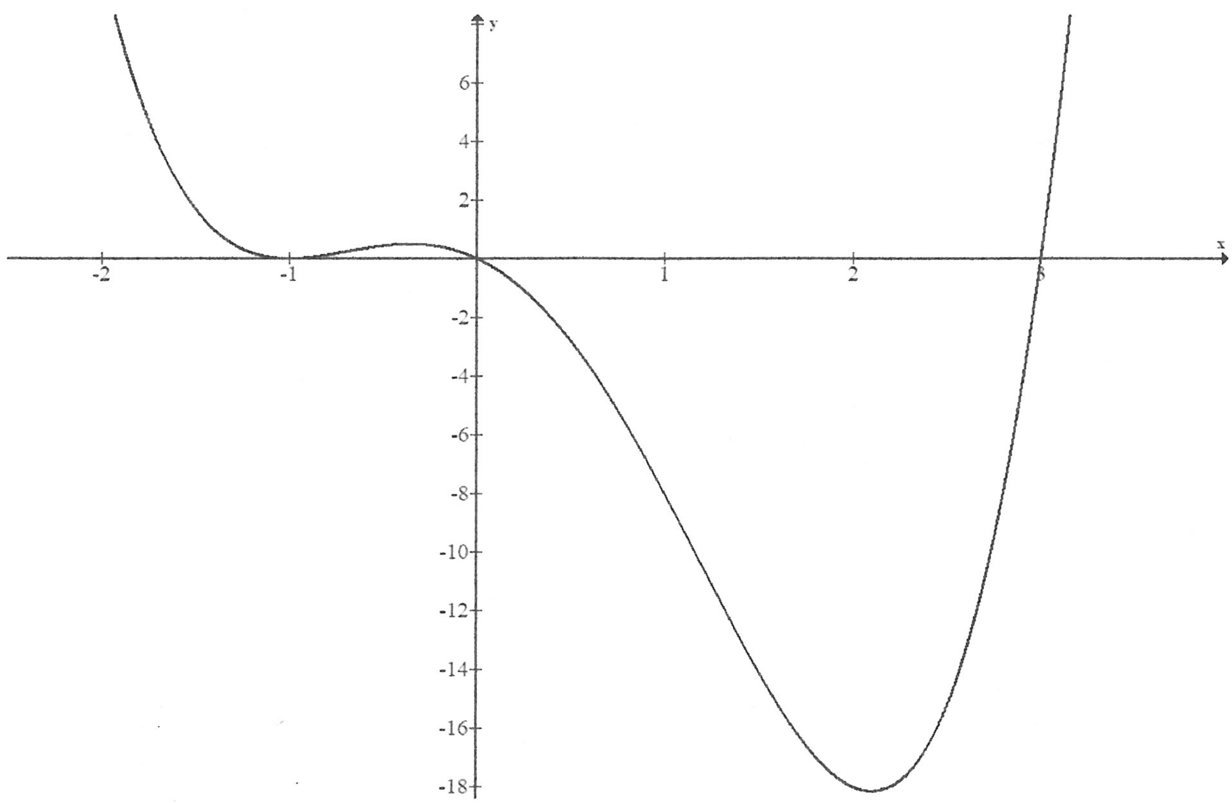
$x = \left(-\frac{1}{8}\right)^{2/3} = \frac{1}{4}$

f concave up:
all x

f concave down: $\quad f''$ $\quad (+) \quad \frac{1}{4} \quad (+)$
N/A

points of inflection:
none

using Second Derivative Test: $\quad x = 0.397$ is a min



The graph above is of the **function** f(x).

- On which x-interval(s) is f(x) concave down?

$\left(-\frac{1}{2}, 1\right)$ (approximately)

- Which x-value(s) is (are) point(s) of inflection in f(x)?

  $x = -\frac{1}{2}$ $x = 1$

Assume now that the graph above is of the **derivative** function $f'(x)$.
With regards to the original function f(x):

- On which x-interval(s) is f(x) increasing?

  $(-2, -1) \cup (-1, 0) \cup (3, 4)$

- On which x-interval(s) is f(x) concave up?

  $(-1, -0.3) \cup (2, 4)$

- At which x-value(s) does f(x) have an inflection point?

  $x = -1$, $x = -0.3$, $x = 2$

Assume now that the **second-derivative** function $f''(x)$ is drawn above.
With regards to the original function f(x):

- At which x-value(s) does f(x) have an inflection point?

  $x = 0, 3$

- On which x-interval(s) is f(x) concave up?

  $(-2, -1) \cup (-1, 0) \cup (3, 4)$