Calculus Study Guide: section 4.6

On a certain farm, the corn yield is:
$Y = -0.118x^2 + 8.5x + 12.9$ (bushels per acre),

where x is the number of corn seeds planted per acre (in thousands). Assume that corn seed costs \$1.25 (per thousand seeds) and that corn can be sold for \$1.50 / bushel.

a) Find the value of x that maximizes yield Y. Then compute the profit (revenue minus the cost of seeds) at planting level x.

$\frac{dY}{dx} = -0.236x + 8.5 = 0$

$x = 36.016$

$Y = 165.972$

profit $= 165.972(1.5) - 36.016(1.25)$

$= 203.938$

b) Compute the profit P(x) as a function of x and find the value of x that maximizes profit.

profit $= (-0.118x^2 + 8.5x + 12.9)(1.5) - 1.25x$

$= -0.177x^2 + 11.5x + 19.35$

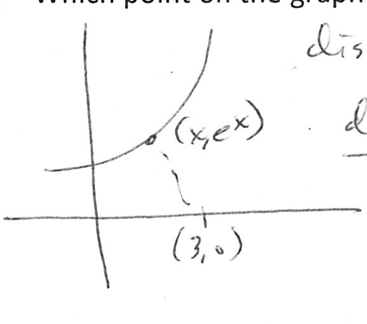
$\frac{dP}{dx} = -0.354x + 11.5 = 0$ $\quad x = 32.485$

$P(32.485) = 206.144$

c) Compare the profit in a) and b). Are they different?

slightly

Which point on the graph of $y = e^x$ is closest to the point (3, 0)?



distance $= \sqrt{(e^x - 0)^2 + (x-3)^2} = (e^{2x} + x^2 - 6x + 9)^{1/2}$

$\frac{d(dist)}{dx} = \frac{1}{2}(e^{2x} + x^2 - 6x + 9)^{-1/2}(2e^{2x} + 2x - 6)$

$\frac{e^{2x} + x - 3}{\sqrt{e^{2x} + x^2 - 6x + 9}} = 0$

set numerator $= 0$ + solve on calculator

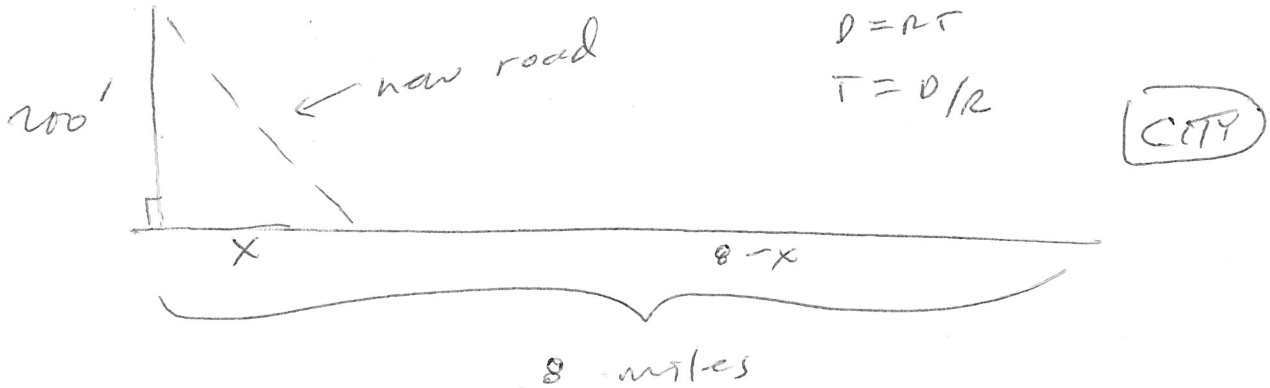
$x = 0.465$

$Y = 1.592$

A new road is being put in that will connect a new subdivision with a highway. The subdivision is 200 feet from the highway. The speed limit on the new road will be 20 mph. If the new road is put in perpendicular to the highway, commuters will drive on the highway for 8 miles to get into the city. The speed limit on the highway is 65 mph. The new road will not actually be put in perpendicular to the highway; it will put in in order to minimize commuters' time into the city. Where should the new road meet the highway; what is the distance between where the perpendicular segment from the subdivision meets the highway and the point where the new road will meet the highway? (1 mile = 5280 feet)

$D = RT$

$T = D/R$



200' = 0.0379 mile

length of new road $= \sqrt{x^2 + 0.00143}$

$$\text{Time} = \frac{\sqrt{x^2+0.00143}}{20} + \frac{8-x}{65}$$

$$\frac{dT}{dx} = \frac{1}{20} \cdot \frac{1}{2}(x^2+0.00143)^{-1/2}(2x) - \frac{1}{65} = 0$$

$$\frac{x}{20\sqrt{x^2+0.00143}} - \frac{1}{65} = 0$$

Solve on calculator.

$x = 0.0122$ miles = 64.4 feet

Viewing $\frac{dT}{dx}$ on the calculator, it is clear from the First Derivative Test that this is a minimum.