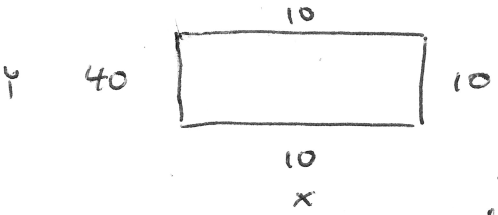
Calculus Study Guide: section 4.6

A landscape architect is planning a fence to go around a rectangular garden. Three of the walls will be chain link fence that costs $10 / foot. The fourth wall will be brick, which costs $40 / foot. The budget for the project is $2500. What should the dimensions of the fence be in order to maximize the garden's area? Then show that these dimensions actually maximize the area.



$\text{Area} = xy$

$40y + 10y + 10x + 10x = 2500$

$50y + 20x = 2500$

$50y = 2500 - 20x$

$y = 50 - 0.4x$

$\text{Area} = x(50 - 0.4x)$

$A = 50x - 0.4x^2$

$\frac{dA}{dx} = 50 - 0.8x = 0$

$x = 62.5$

$y = 50 - 0.4(62.5) = 25$

Show that this gives a max: $A'' = -0.8 < 0$

$\therefore$ it's a max

74. The area of the shaded region in Figure 5 is $\frac{1}{2}r^2(\theta - \sin\theta)$. What is the maximum possible area of this region if the length of the circular arc is 1?

$r\theta = 1$

$r = 1/\theta$

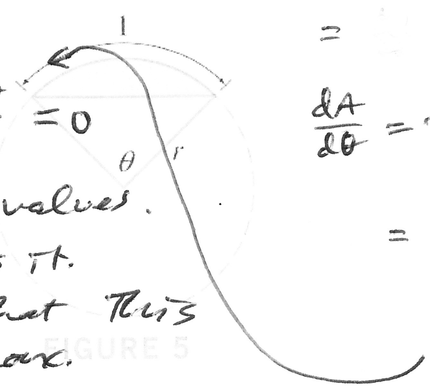
$\text{Area} = \frac{1}{2}\left(\frac{1}{\theta}\right)^2(\theta - \sin\theta)$

$= \frac{\theta - \sin\theta}{2\theta^2}$

$\frac{dA}{d\theta} = \frac{2\theta^2(1 - \cos\theta) - (\theta - \sin\theta)\cdot 4\theta}{4\theta^4}$

$= \frac{2\theta^2 - 2\theta^2\cos\theta - 4\theta^2 + 4\theta\sin\theta}{4\theta^4}$

$\frac{-2\theta - 2\theta\cos\theta + 4\sin\theta}{4\theta^3} = 0$



$\frac{2\sin\theta - \theta - \theta\cos\theta}{2\theta^3} = 0$

Try some simple values.

$\theta = \pi$ solves it.

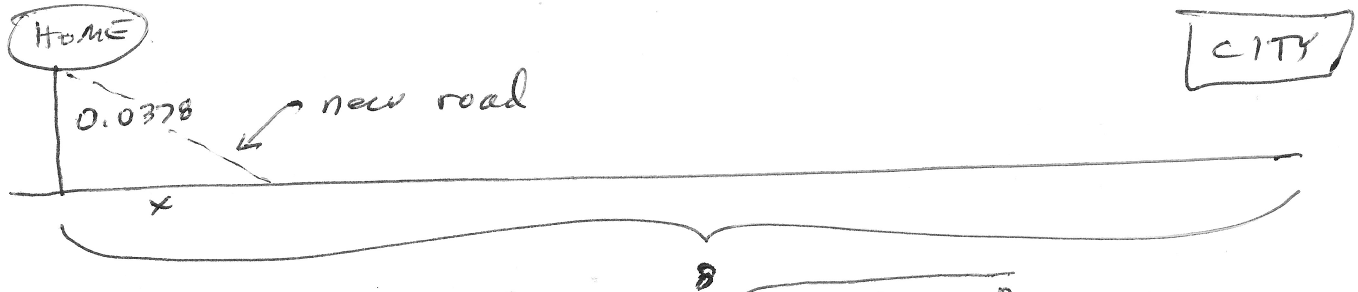
It's apparent that this is a max.

$\text{Area} = \frac{1}{2\pi}$

Recall that the formula for the length of the circular arc = rθ. So here, a constraint is rθ = 1.

A new road is being put in that will connect a new subdivision with a highway. The subdivision is 200 feet from the highway. The speed limit on the new road will be 20 mph. If the new road is put in perpendicular to the highway, commuters will drive on the highway for 8 miles to get into the city. The speed limit on the highway is 65 mph. The new road will not actually be put in perpendicular to the highway; it will put in in order to minimize commuters' time into the city. Where should the new road meet the highway; what is the distance between where the perpendicular segment from the subdivision meets the highway and the point where the new road will meet the highway? (1 mile = 5280 feet)

$200' = 0.0378$ mile



length of new road $= \sqrt{0.0378^2 + x^2}$

$D = RT \qquad T = D/R$

time on new road $= (0.00143 + x^2)^{1/2} / 20$

time on highway $= (8 - x)/65$

Total time $= \dfrac{(0.00143 + x^2)^{1/2}}{20} + \dfrac{8 - x}{65}$

$$\frac{dT}{dx} = \frac{\frac{1}{2}(0.00143 + x^2)^{-1/2}(2x)}{20} - \frac{1}{65} = 0$$

$$\frac{x}{20\sqrt{0.00143 + x^2}} - \frac{1}{65} = 0$$ solve on Calculator

$x = 0.0122$ miles $= 64.4$ feet

The graph of $\frac{dT}{dx}$ shows the graph crossing from negative to positive at $x = 0.0122$ miles.
By the First Derivative Test, This is a min.