Calculus Study Guide: section 4.6

On a certain farm, the corn yield is:
$Y = -0.118x^2 + 8.5x + 12.9$ (bushels per acre),

where x is the number of corn plants per acre (in thousands). Assume that corn seed costs $1.25 (per thousand seeds) and that corn can be sold for $1.50 / bushel.

a) Find the value x_0 that maximizes yield Y. Then compute the profit (revenue minus the cost of seeds) at planting level x_0.

$\frac{dY}{dx} = -0.236x + 8.5 = 0 \qquad x = 36.016 \qquad Y = 165.972$

$\text{Profit} = 165.972(1.5) - 36.016(1.25) = 203.938$

b) Compute the profit P(x) as a function of x and find the value x_1 that maximizes profit.

$\text{Profit} = (-0.118x^2 + 8.5x + 12.9)(1.5) - 1.25x$

$= -0.177x^2 + 11.5x + 19.35$

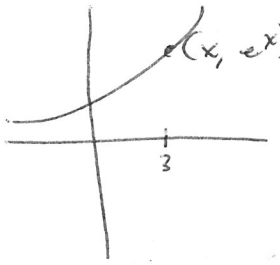
$\frac{dP}{dx} = -0.354x + 11.5 = 0 \qquad x = 32.485$

$P(32.485) = 206.144$

c) Compare the profit at levels x_0 and x_1. Are they different?

slightly

Which point on the graph of $y = e^x$ is closest to the point (3, 0)?



$(x-3)^2 + (e^x - 0)^2 = D^2 = x^2 - 6x + 9 + e^{2x}$

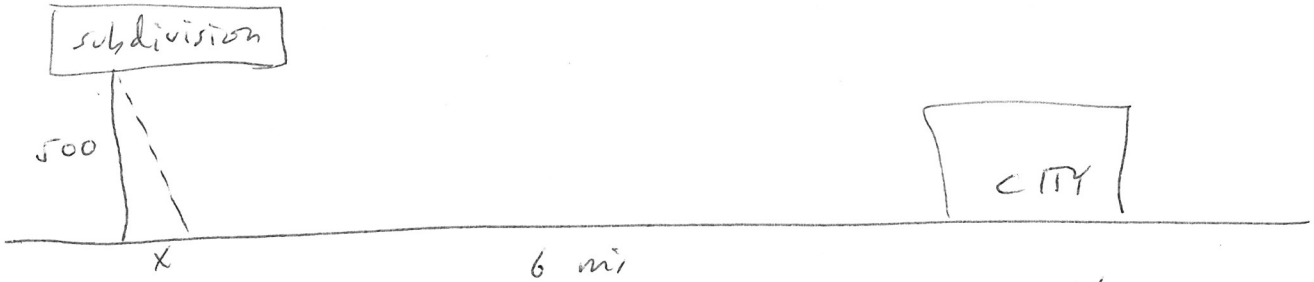
$\frac{d(D^2)}{dx} = 2x - 6 + 2e^{2x} = 0$

(solve on calculator)

$x = 0.465$

$y = e^{0.465} = 1.592$

A new road is being put in that will connect a new subdivision with a highway. The subdivision is 500 feet from the highway. The speed limit on the new road will be 30 mph. If the new road is put in perpendicular to the highway, commuters will drive on the highway for 6 miles to get into the city. The speed limit on the highway is 55 mph. The new road will not actually be put in perpendicular to the highway; it will put in in order to minimize commuters' time into the city. Where should the new road meet the highway; what is the distance between where the perpendicular segment from the subdivision meets the highway and the point where the new road will meet the highway?



Convert 500 ft to miles = 0.0947 miles

length of new road = $\sqrt{0.0947^2 + x^2}$

$D = RT$ time on new road = $\frac{\sqrt{0.0947^2 + x^2}}{30}$

length of highway = $6 - x$

time on highway = $\frac{6-x}{55}$

T = total time = $\frac{\sqrt{0.00897 + x^2}}{30} + \frac{6-x}{55}$

$\frac{dT}{dx} = \frac{\frac{1}{2}(0.00897 + x^2)^{-1/2} \cdot 2x}{30} - \frac{1}{55} = 0$

This is an equation - solve on calculator.

$x = 0.0616$ miles $= 325$ feet