

Calculus Study Guide: section 4.7

Verify that L'Hopital's Rule applies and evaluate the limit.

$$\lim_{x \rightarrow 0^+} \sqrt{x} \ln x \quad 0 \cdot -\infty \quad \text{rewrite it} \quad \frac{\ln x}{\frac{1}{x^{1/2}}} \xrightarrow{LH} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = -2x^{1/2} \rightarrow 0$$

$$\lim_{t \rightarrow \infty} \frac{\ln(e^t + 1)}{t} \quad \frac{\infty}{\infty} \xrightarrow{LH} \frac{1}{e^t + 1} \cdot e^t = 1$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{1-x^2}}{\cos^{-1}x} \quad \frac{0}{0} \xrightarrow{LH} \frac{\frac{1}{2}(1-x^2)^{-1/2}(-2x)}{-\frac{1}{\sqrt{1-x^2}}} = x = 1$$

Which function grows faster, $f(x) = x^2$ or $g(x) = 2^x$?

$$\frac{x^2}{2^x} \quad \frac{\infty}{\infty} \xrightarrow{LH} \frac{2x}{2^x \ln 2} \quad \frac{\infty}{\infty} \xrightarrow{LH} \frac{2}{2^x (\ln 2)^2} \rightarrow 0 \quad 2^x \gg x^2$$