

Study Guide: 5.8

A differential equation is: $\frac{dy}{dt} = -2y$. It is given that $y(0) = 12$.

What is the general solution? $y = y_0 e^{-2t}$

What is the particular solution? $y = 12e^{-2t}$

The radioactive element Bulldogium has a half-life of 20.18 days. After 90 days, only 1 g remains. How many grams of Bulldogium were present initially?

$$k = \frac{\ln 2}{\text{half-life}} = \frac{\ln 2}{20.18} = 0.0343 \quad B = B_0 e^{-0.0343 t}$$

$$1 = B_0 e^{-0.0343(90)} = B_0 (0.0456) \quad B_0 = \frac{1}{0.0456} = 21.911$$

Initial grams = 21.911

A population of bacteria in a lab measures 1000 at time $t = 0$. Its doubling time is 1 hour.

A second population measures 3000 at $t = 0$. Its doubling time is 2 hours.

At about what time will the two populations be the same size?

$$1000 \cdot 2^t = 3000 \cdot 2^{t/2}$$

$$\frac{2^t}{2^{t/2}} = 3$$

$$2^{t/2} = 3$$

$$\ln 2^{t/2} = \ln 3$$

$$t/2 \cdot \ln 2 = \ln 3$$

$$t = 2 \cdot \frac{\ln 3}{\ln 2} = 3.169$$

3.169 hours

answer: _____