Study Guide: 6.2

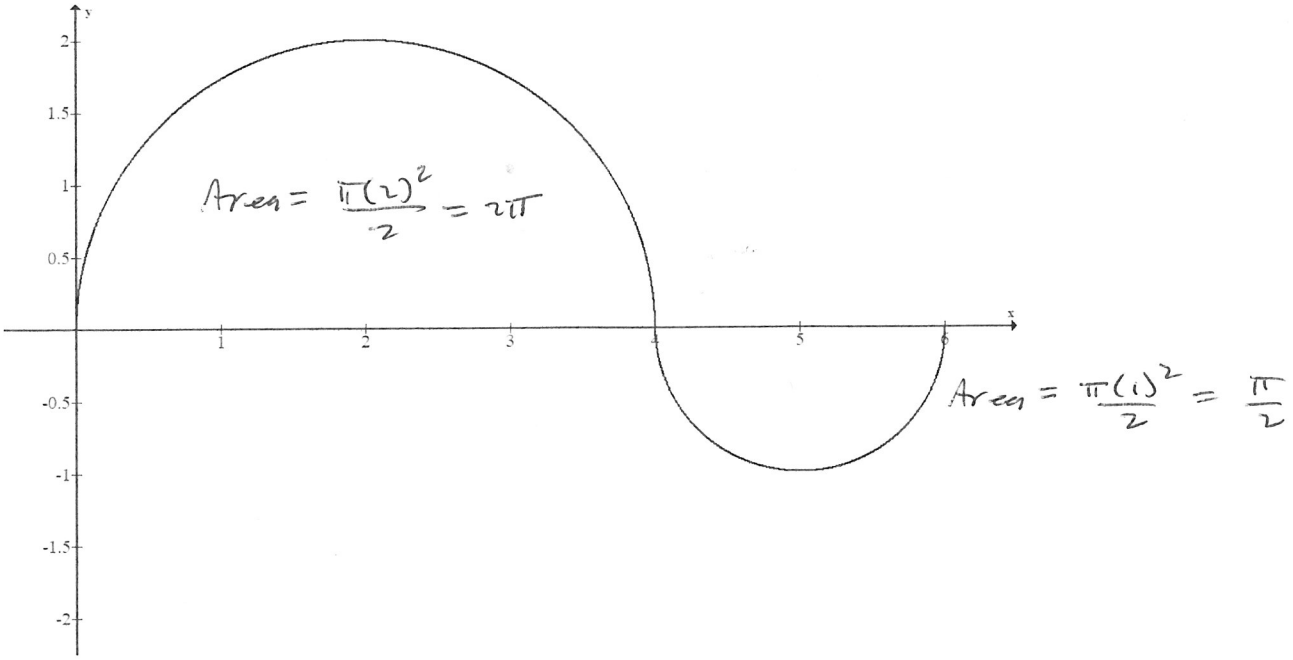
Non-Calculator

What is the average value of h(t) = $\sec^2 x$ on the interval $(-\frac{\pi}{3}, \frac{\pi}{6})$?

$$\frac{1}{\frac{\pi}{6} - \left(-\frac{\pi}{3}\right)} \int_{-\pi/3}^{\pi/6} \sec^2 x \, dx = \frac{1}{\frac{\pi}{2}} \cdot \tan x \Big|_{-\pi/3}^{\pi/6} = \frac{2}{\pi}\left(\tan\frac{\pi}{6} - \tan\frac{-\pi}{3}\right)$$

$$= \frac{2}{\pi}\left(\frac{1}{\sqrt{3}} - -\sqrt{3}\right) = \frac{2}{\pi}\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$$

answer: _____



The graph above – which has 2 semicircles - shows the velocity of an object over a 6-minute period. The units on the y-axis are ft/min.

1. What is the average velocity of the object?

$$\frac{1}{6}\int v(t)\,dt = \frac{1}{6}\left(2\pi - \frac{\pi}{2}\right) = \frac{1}{6}\left(\frac{3\pi}{2}\right) = \frac{\pi}{4}$$

2. What is the average speed of the object?

$$\frac{1}{6}\int |v|\,dt = \frac{1}{6}\left(2\pi + \frac{\pi}{2}\right) = \frac{1}{6}\left(\frac{5\pi}{2}\right) = \frac{5\pi}{12}$$

Calculator-Active

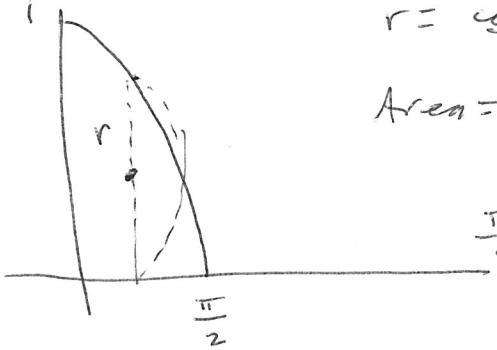
If the average temperature in a school is expected to exceed 90° during a school day, school will be cancelled. The school day runs from 8 a.m. (t = 8) to 3 p.m. (t = 15). The temperature is given by $T(t) = 95 - 0.4(t - 14)^2$. Should school be held or cancelled this day?

$$\frac{1}{7}\int_{8}^{15} 95 - 0.4(t-14)^2\,dt = 90.866$$

Hold school

Cancel school

The base of a solid is the area enclosed by y = cos x, the y-axis and the x-axis from x = 0 to x = π/2. The cross-sections perpendicular to the x-axis are semicircles. Find the solid's volume.



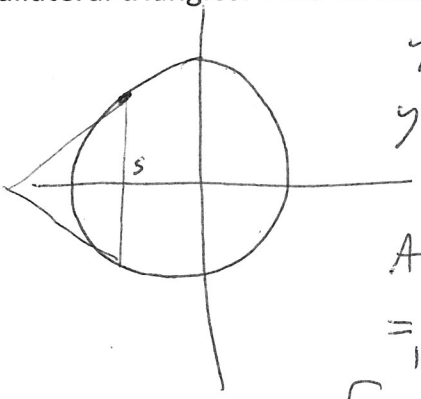
$$r = \frac{\cos x}{2}$$

$$\text{Area} = \frac{\pi}{2}\left(\frac{\cos x}{2}\right)^2 = \frac{\pi \cos^2 x}{8}$$

$$\frac{\pi}{8}\int_{0}^{\pi/2} \cos^2 x\,dx = 0.0981\pi$$

$$= 0.308$$

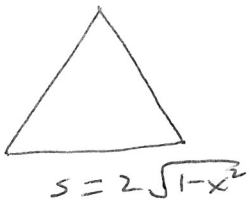
The base of a solid is the unit circle $x^2 + y^2 = 1$, and its cross-sections perpendicular to the x-axis are equilateral triangles. Find its volume.



$y^2 = 1 - x^2$

$y = \sqrt{1-x^2}$

$s = 2\sqrt{1-x^2}$



$s = 2\sqrt{1-x^2}$

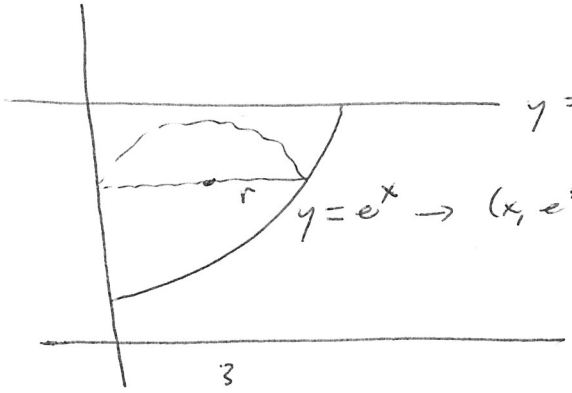
Area of $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}\left(2\sqrt{1-x^2}\right)^2 \cdot \frac{\sqrt{3}}{2}$

$= \sqrt{3}(1-x^2)$

$$\sqrt{3}\int_{-1}^{1} 1-x^2\,dx = 2.309$$

The base of a solid is the region enclosed by the graphs of $y = e^x$, $y = 3$ and the y-axis. Cross-sections perpendicular to the y-axis are semicircles. Find its volume.

$\ln y = x$



$(\ln y, y)$ $\quad r = \frac{\ln y}{2}$

Area $= \frac{\pi}{2}\left(\frac{\ln y}{2}\right)^2 = \frac{\pi \cdot (\ln y)^2}{8}$

$$\frac{\pi}{8}\int_{1}^{3} (\ln y)^2\,dy = 0.128\pi$$

$= 0.404$