Calculus Study Guide: 7.2

Evaluate the integrals.

$$\int_0^{\frac{\pi}{4}} x\sec^2 x\, dx$$

$u = x \qquad dv = \sec^2 x$

$du = dx \qquad v = \tan x$

$= x\tan x - \int \tan x\, dx$

$= x\tan x - \ln|\sec x| \Big|_0^{\pi/4}$

$= \frac{\pi}{4}\cdot 1 - \ln(\sec \pi/4) - (0 - \ln(\sec 0))$

$= \frac{\pi}{4} - \ln\sqrt{2} + \ln(1) \quad = \frac{\pi}{4} - \ln\sqrt{2}$

answer: ______________________________

$\int x^3 \ln(x)\, dx$

$u = \ln x \qquad dv = x^3$

$du = \frac{1}{x} \qquad v = \frac{1}{4}x^4$

$= \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\, dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\, dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\cdot\frac{1}{4}x^4$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

answer: ______________________________

$\int e^{2x} \sin(x)\, dx$

$u = e^{2x}$ $\quad dv = \sin x\, dx$

$du = 2e^{2x}$ $\quad v = -\cos x$

$= -e^{2x} \cos x - \int -\cos x (2e^{2x}) dx$

$= -e^{2x} \cos x + \underbrace{2\int e^{2x} \cos x\, dx}_{\text{Do I.B.P. on This}}$

$\int e^{2x} \cos x\, dx$

$u = e^{2x}$ $\quad dv = \cos x\, dx$

$du = 2e^{2x} dx$ $\quad v = \sin x$

$= e^{2x} \sin x - 2\int \sin x\, e^{2x}\, dx$

$\int e^{2x} \sin(x)\, dx = -e^{2x} \cos x + 2\left[\left(e^{2x} \sin x\right) - 2\int \sin x\, e^{2x}\, dx\right]$

$= -e^{2x} \cos x + 2e^{2x} \sin x - 4\int e^{2x} \sin x\, dx$

$5\int e^{2x} \sin x\, dx = -e^{2x} \cos x + 2e^{2x} \sin x$

$\int e^{2x} \sin x\, dx = \frac{-e^{2x} \cos x + 2e^{2x} \sin x}{5}$

answer: __________________________________

$\int_0^1 x^3 3^x dx$

Use tabular integration.

u	dv
x^3	3^x
$3x^2$	$3^x/\ln 3$
$6x$	$3^x/(\ln 3)^2$
6	$3^x/((\ln 3)^3$
0	$3^x/((\ln 3)^4$

$\frac{x^3 3^x}{\ln 3} - \frac{3x^2 3^x}{(\ln 3)^2} + \frac{6x\, 3^x}{(\ln 3)^3} - \frac{6 \cdot 3^x}{(\ln 3)^4}$

answer: __________________________________