Calculus Study Guide: 7.3

Trig Identities:

$\sin(m+n) = \sin m \cos n + \cos m \sin n$ $\qquad$ $\sin(m-n) = \sin m \cos n - \cos m \sin n$

$\cos(m + n) = \cos m \cos n - \sin m \sin n$ $\qquad$ $\cos(m - n) = \cos m \cos n + \sin m \sin n$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \dfrac{1+\cos(2x)}{2}$ $\qquad$ $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

Evaluate the integrals.

$\int \cos^4 x\, dx \rightarrow \cos^2 x \cos^2 x = \dfrac{1+\cos(2x)}{2} \cdot \dfrac{1+\cos 2x}{2} = \frac{1}{4}\left(1 + 2\cos 2x + \cos^2 2x\right)$

$\cos^2 2x = \dfrac{1+\cos 4x}{2}$ $\quad$ substitute: $\frac{1}{4}\left(1 + 2\cos 2x + \dfrac{1+\cos 4x}{2}\right)$

$= \frac{1}{4}\left(1.5 + 2\cos 2x + \dfrac{\cos 4x}{2}\right) \xrightarrow{\text{Integrate}} \frac{1}{4}\left(1.5x + \sin 2x + \frac{1}{8}\sin 4x\right)$

$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

$\int \sin^3 x \cos^2 x\, dx \rightarrow \sin^2 x \cos^2 x \sin x\, dx \rightarrow (1-\cos^2 x)\cos^2 x \sin x\, dx$

$(\cos^2 x - \cos^4 x)\sin x\, dx$ $\quad$ Let $u = \cos x$ $\quad$ $du = -\sin x\, dx$

Rewrite: $-(u^2 - u^4)\, du$ $\quad$ $\int u^4 - u^2\, du = \frac{1}{5}u^5 - \frac{1}{3}u^3$

$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

$\int \sin(2x)\cos(2x)dx$ Add together the sum + difference identities for the sine. $\quad \sin(m+n) + \sin(m-n) = 2\sin m \cos n$

$\int \sin 2x \cos 2x\, dx = \int \frac{1}{2}\left[\sin 4x + \sin 0x\right] dx = \frac{1}{2}\int \sin 4x\, dx$

$= -\frac{1}{2} \cdot \dfrac{\cos 4x}{4} = -\frac{1}{8}\cos 4x + C$