

Calculus Study Guide: 7.6

Evaluate the integrals.

$$\int_0^1 \frac{2x+1}{x^2-7x+12} dx \quad \frac{2x+1}{x^2-7x+12} = \frac{A}{x-4} + \frac{B}{x-3}$$

$$2x+1 = A(x-3) + B(x-4)$$

$$\text{Let } x=3: 7 = -B \quad B = -7$$

$$\text{Let } x=4: 9 = A$$

$$9 \int \frac{dx}{x-4} + -7 \int \frac{dx}{x-3} = 9 \ln|x-4| - 7 \ln|x-3|$$

$$9 \ln|x-4| - 7 \ln|x-3| \Big|_0^1 = 9 \ln|1-4| - 7 \ln|1-3|$$

$$- (9 \ln|0-4| - 7 \ln|0-3|) = 9 \ln 3 - 7 \ln 2$$

$$- 9 \ln 4 + 7 \ln 3$$

$$= 16 \ln 3 - 9 \ln 4 - 7 \ln 2$$

$$\int \frac{3x^2-10}{x^2-4x+4} dx \quad \text{First, divide.}$$

$$\begin{array}{r} x^2-4x+4 \overline{) 3x^2-10} \\ \underline{3x^2-12x+12} \\ 12x-22 \end{array}$$

$$3 + \frac{12x-22}{x^2-4x+4}$$

$$\frac{12x-22}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$12x-22 = A(x-2) + B$$

$$\text{Let } x=2: 2 = B$$

$$\text{Let } x=3: 14 = A+B = A+2 \quad \therefore A=12$$

$$\int \frac{3x^2-10}{x^2-4x+4} dx = \int 3 dx + 12 \int \frac{dx}{x-2} + 2 \int \frac{dx}{(x-2)^2}$$

$$= 3x + 12 \ln|x-2| + 2(- (x-2)^{-1})$$

$$= 3x + 12 \ln|x-2| - \frac{2}{x-2} + C$$