Calculus Study Guide: 8.4

Find the order-3 Maclaurin polynomial T3(x) for g(x) = $\sqrt{x+1}$.

$g(0) = 1$

$g'(x) = \frac{1}{2}(x+1)^{-1/2} = \frac{1}{2\sqrt{x+1}}$ $\quad g'(0) = \frac{1}{2}$

$g''(x) = -\frac{1}{4}(x+1)^{-3/2} = -\frac{1}{4(x+1)^{3/2}} = -\frac{1}{4}$

$g'''(x) = \frac{3}{8}(x+1)^{-5/2} = \frac{3}{8(x+1)^{5/2}} = \frac{3}{8}$

$T_3(x) = 1 + \frac{1}{2}x - \frac{\frac{1}{4}x^2}{2!} + \frac{\frac{3}{8}x^3}{3!}$

$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

Use T3(x) to find an order-3 Maclaurin polynomial that approximates k(x) = $\sqrt{3x+1}$.

$1 + \frac{1}{2}(3x) - \frac{1}{8}(3x)^2 + \frac{1}{16}(3x)^3$

$1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3$

Use T3(x) to find a Maclaurin polynomial that approximates m(x) = $x^3\sqrt{x+1}$.

$x^3\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3\right)$

$= x^3 + \frac{1}{2}x^4 - \frac{1}{8}x^5 + \frac{1}{16}x^6 + \cdots$

Use T3(x) to estimate $\sqrt{e+1}$.

$\sqrt{e+1} \approx 1 + \frac{1}{2}(e) - \frac{1}{8}e^2 + \frac{1}{16}e^3 = 2.690$

$\sqrt{e+1} \approx 1.928$

Use $T_3(x)$ to find a order-4 Maclaurin polynomial that approximates $h(x) = \frac{2}{3}(x+1)^{\frac{3}{2}}$.
(The $\frac{3}{2}$ is in the exponent.)

$$\int (x+1)^{1/2}\,dx = \frac{2}{3}(x+1)^{3/2} = \int 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3\,dx$$

$$x + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{64}x^4 + c \qquad (c = 2/3)$$

$$\frac{2}{3} + x + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{64}x^4$$

Use $T_3(x)$ to find an order-2 Maclaurin polynomial that approximates $i(x) = \frac{1}{2\sqrt{x+1}}$.

$$\frac{d}{dx} g(x) = \frac{d}{dx}(x+1) = \frac{1}{2\sqrt{x+1}}$$

$$\frac{d}{dx}\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3\right) = \frac{1}{2} - \frac{1}{4}x + \frac{3}{16}x^2$$

Find the $T_3(x)$ Taylor polynomial with center a = 3 that approximates $j(x) = \sqrt{x+1}$.

$$j(3) = 2$$

$$j'(x) = \frac{1}{2}(x+1)^{-1/2} \qquad j'(3) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$j''(x) = -\frac{1}{4}(x+1)^{-3/2} \qquad j'(3) = -\frac{1}{4 \cdot 4^{3/2}} = -\frac{1}{32}$$

$$j'''(x) = \frac{3}{8}(x+1)^{-5/2} \qquad j'''(3) = \frac{3}{8(4)^{5/2}} = \frac{3}{256}$$

$$T_3(x) = 2 + \frac{1}{4}(x-3) - \frac{1}{32}\frac{(x-3)^2}{2!} + \frac{3}{256}\frac{(x-3)^3}{3!}$$