Calculus Study Guide: 9.1

Solve the initial-value problems.

$y'(1+x^2) = \sec y$ where y(1) = 0

$$\cos y \, dy = \frac{dx}{1+x^2}$$

$$\int \cos y \, dy = \int \frac{dx}{1+x^2}$$

$$\sin y = \tan^{-1} x + c$$

$$y = \sin^{-1}(\tan^{-1} x + c)$$

$$0 = \sin^{-1}(\tan^{-1} 1 + c)$$

$$0 = \sin^{-1}(\pi/4 + c)$$

$$c = -\pi/4$$

$$y = \sin^{-1}(\tan^{-1} x - \pi/4)$$

general solution: ______________________________

particular solution: ______________________________

$\frac{dy}{dx} = e^{0.5x}\sqrt{1-y^2}$ where y(2) = 1

$$\frac{dy}{\sqrt{1-y^2}} = e^{0.5x} dx$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int e^{0.5x} dx$$

$$\sin^{-1} y = 2e^{\frac{1}{2}x} + c$$

$$y = \sin(2e^{\frac{1}{2}x} + c)$$

$$1 = \sin(2e + c)$$

$$2e + c = \pi/2$$

$$c = \frac{\pi}{2} - 2e$$

$$y = \sin\left(2e^{\frac{1}{2}x} + \frac{\pi}{2} - 2e\right)$$

general solution: ______________________________

particular solution: ______________________________

We derived the differential equation $\frac{dy}{dt} = \frac{Bv(y)}{A(y)}$ in class to describe the height y of liquid in a tank, with a small hole in the bottom of the tank.

B = area of hole

A = cross-sectional area of tank

v = velocity of water leaving the tank through the hole

To get v(y), we have Torricelli's Law: $v(y) = -\sqrt{2gy} = -8\sqrt{y}$ [since g = 32 in English units]

A given tank is a cylinder with a radius of 3 ft. It is 12 feet high. The hole in the bottom of the tank is 1 square inch.

$1 \text{ in}^2 = \frac{1}{144} \text{ ft}^2$

The tank is initially full of water. How long does it take for the tank to lose its water?

$$\frac{dy}{dt} = \frac{\frac{1}{144}(-8\sqrt{y})}{9\pi} = -0.0196\, y^{1/2}$$

$$\int y^{-1/2}\, dy = -\int 0.00196\, dt$$

$$2y^{1/2} = -0.00196t + c$$

$$y^{1/2} = -0.000982t + c$$

$$y = (-0.000982t + c)^2$$

$$12 = c^2$$

$$c = \sqrt{12}$$

$$y = (-0.000982t + \sqrt{12})^2$$

$$0 = (-0.000982t + \sqrt{12})^2$$

$$0 = -0.000982t + \sqrt{12}$$

answer: ________________________________

$$t = \frac{\sqrt{12}}{0.000982} = 3527 \text{ secs}$$

58.8 minutes