Assume that Newton's Law of Cooling applies here. The relevant differential equation is: $\frac{dT}{dt} = k(T - T_A)$,

where T is temperature, t is time and T_A is the ambient temperature.

A beverage at 33° is brought into a large room with ambient temperature 70°. After 5 minutes, the beverage's temperature T is 40°.

What will the beverage's temperature be after 10 minutes?

$T = T_A + (T_0 - T_A)e^{-kt}$ $\quad T(0) = T_0$

$T(\infty) = T_A$ $\quad T = 70 + (33-70)e^{-kt}$

$40 = 70 - 37e^{-5k}$ $\quad e^{-5k} = 0.811$ $\quad -5k = \ln 0.811$

$-5k = -0.209$ $\quad k = 0.0419$

$T(10) = 70 - 37e^{-0.0419(10)} = 45.665$

T(10) = 45.665°

How many minutes does it take the beverage's temperature to rise from 33° to 60°?

$60 = 70 - 37e^{-0.0419t}$ $\quad 0.270 = e^{-0.0419t}$

$-0.0419t = -1.309$ $\quad t = 31.249$ minutes

answer = __________

For a skydiver in free fall, we derived the differential equation:

$$\frac{dv}{dt} = -\frac{k}{m}\left(v + \frac{gm}{k}\right) \quad [1]$$

which applies if the units are metric, and where v is the skydiver's velocity (m/s), t is time (s), m is the skydiver's mass (kg), g = 9.8 m/s^2, and k is a constant that depends on the aerodynamic qualities of the skydiver.

In section 9.2 we worked with the differential equation $\frac{dt}{dt} = k(y - b)$ which has the solution y(t) = b + Ce^{kt}.

Solve [1]. $v = -\frac{gm}{k} + Ce^{-\frac{k}{m}t}$

A 100-kg skydiver jumps out of an airplane (with zero initial velocity).

What is their terminal velocity if k = 10 kg/s?

$v(0) = 0 \quad v(0) = 0 = -\frac{gm}{k} + Ce^0 \quad C = gm/k$

$v(t) = -\frac{gm}{k} + \frac{gm}{k}e^{-kt/m} \quad \text{terminal } v = -\frac{gm}{k}$

$\text{terminal } v = -\frac{9.8(100)}{10} = -98 \text{ m/s}$

What is their velocity after 3 seconds of falling?

$v(t) = -98 + 98e^{-0.1t}$

$v(3) = -98 + 98e^{-0.3} = -25.399 \text{ m/s}$

After s/he reaches terminal velocity, s/he opens the parachute and k increases to 100. At what velocity will s/he land on the ground?

$\text{terminal velocity} = -\frac{gm}{k} = -\frac{9.8(100)}{100}$

$= -9.8 \text{ m/s}$